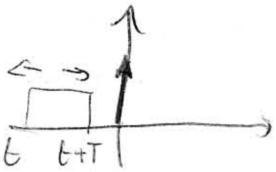
19/11/07

① a) $y(t) = -\int_t^{t+T} x(\tau)\,d\tau \qquad T>0$ mando in ingresso $\delta(\tau)$

$y(t) = -\int_t^{t+T} \delta(\tau)\,d\tau$



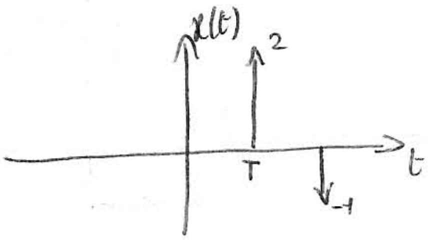
$h(t) = \begin{cases} 0 & \text{per } t+T<0 \\ -1 & \text{per } t<0,\ t+T>0 \\ 0 & \text{per } t>0 \end{cases}$

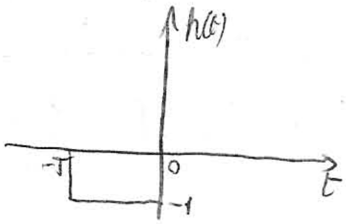
$h(t) = \begin{cases} -1 & \text{per } 0>t>-T \\ 0 & \text{altrove} \end{cases}$

Segnale h(t) non causale perché $h(t) \neq 0$ per $t<0 \Rightarrow$ Sistema non causale

Stabilità: $\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{-T}^{0} +1\,dt = \left[\,t\,\right]_{-T}^{0} = (0+T) = +T \Rightarrow$ stabile B.I.B.O. per $T \neq \pm\infty$

b) $x(t) = 2\delta(t-T) - \delta(t-2T)$

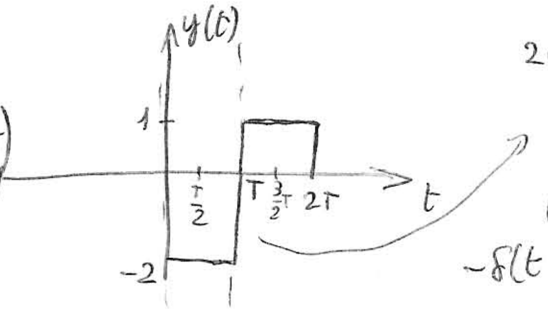




$y(t) = x(t) * h(t)$

Facendo entrare una δ, il sistema porta in uscita $h(t)$ traslata della δ e amplificata. Quindi:

$y(t) = -2\,\Pi\left(\frac{t-T/2}{T}\right) + \Pi\left(\frac{t-\frac{3}{2}T}{T}\right)$

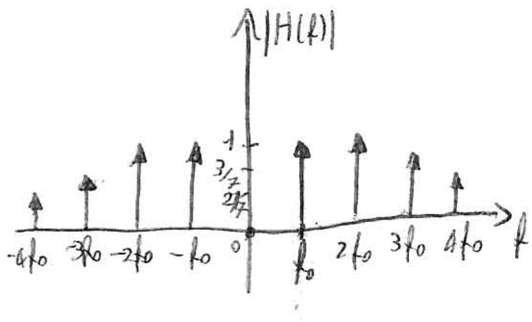


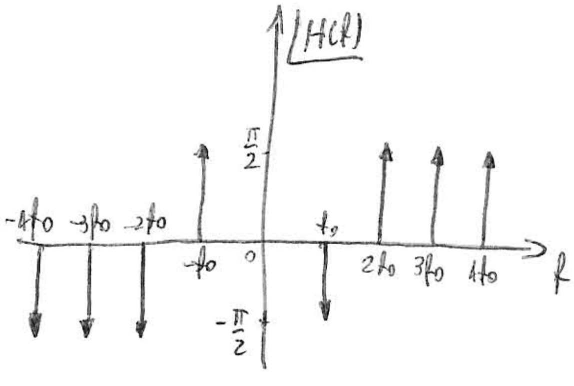
$2\delta(t-T)$: traslo a destra $h(t)$ di T e la moltiplico per 2 (solo per sistemi L.T.I.)

$-\delta(t-2T)$: traslo a destra $h(t)$ di $2T$ e ribalto.

② a) $X_k = \frac{jk}{k^2-2}$ $\qquad X_0 = 0 \quad X_1 = -j \quad X_2 = j \quad X_3 = \frac{3}{7}j \quad X_4 = \frac{2}{7}j$

$X_{-1} = j \quad X_{-2} = -j \quad X_{-3} = -\frac{3}{7}j \quad X_{-4} = -\frac{2}{7}j$





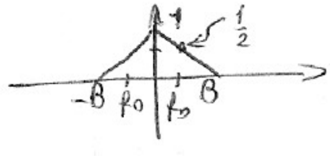
b) $B = 4000\ Hz$ 1) Per essere una sinusoide devo avere una sola armonica per f_0. Il filtro fa passare le armoniche da $-B$ a B, pertanto voglio che le due righe stiano tra $-B$ e B! Dato che il grafico è pari, considero solo il piano di $f > 0$ e impongo $f_0 < B < 2f_0$

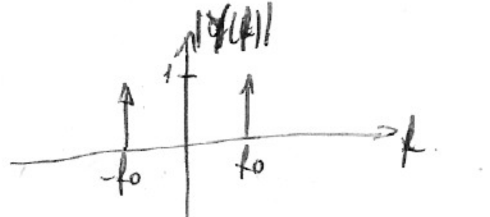
$$\begin{cases} f_0 < B \\ 2f_0 > B \end{cases} \quad \begin{cases} f_0 < 4000\ Hz \\ f_0 > 2000\ Hz \end{cases}$$

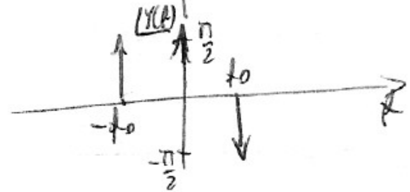
voglio la prima: $f_0 = 2 kHz$.

2) Il segnale di uscita sarà ovviamente una sinusoide.

$$\underset{4000 Hz}{\overset{\shortparallel}{B}} = 2 \underset{2000 Hz}{\overset{\shortparallel}{f_0}}$$







L'ampiezza verrà moltiplicata per $\frac{1}{2}$, mentre la fase verrà sommata a $\pm \frac{\pi}{2}$.

L'uscita sarà quindi

$A = 1 \cdot \frac{1}{2} = \frac{1}{2}$ $\quad \varphi = \pm \frac{\pi}{2} \mp \frac{\pi}{2} = 0$

$y(t) = A \cos(2\pi f_0 t + \varphi)$

$y(t) = \frac{1}{2} \cos(2\pi \cdot 2000 \cdot t) = \frac{1}{2} \cos(4000 \pi t)$