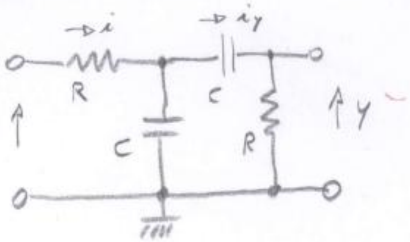
Tracce delle soluzioni

A1.



$$Y = R\, I_Y$$

$$I_Y = I \cdot \frac{\frac{1}{sC}}{\frac{1}{sC} + \frac{1}{sC} + R} = I \cdot \frac{1}{2 + RCs}$$

$$I = \frac{U}{R + \frac{\frac{1}{sC}\cdot\left(\frac{1}{sC} + R\right)}{\frac{1}{sC} + \frac{1}{sC} + R}} = \frac{U}{R + \frac{R + \frac{1}{sC}}{2 + RCs}}$$

$$Y = R \frac{U}{R + \frac{R + \frac{1}{sC}}{2+RCs}} \cdot \frac{1}{2+RCs} = \frac{U}{1 + \frac{1 + \frac{1}{sCR}}{2+RCs}} \cdot \frac{1}{2+RCs}$$

$$G(s) = \frac{1}{2 + RCs + 1 + \frac{1}{RCs}} = \boxed{\frac{RCs}{(RC)^2 s^2 + 3RCs + 1}} \quad \text{f.d.t.}$$

eq. diff. $\quad (RC)^2 D^2 y(t) + 3RC\, Dy(t) + y(t) = RC\, Du(t)$

zeri : $\quad z_1 = 0 \qquad$ poli : $\quad P_{1,2} = \dfrac{-3RC \pm \sqrt{9(RC)^2 - 4(RC)^2}}{2(RC)^2} =$

$$= \frac{-3 \pm \sqrt{5}}{2RC}$$

modi : $\left\{ \exp\left(\dfrac{-3+\sqrt{5}}{2RC}\right),\ \exp\left(\dfrac{-3-\sqrt{5}}{2RC}\right) \right\}$

guadagno statico : $\quad G(0) = 0.$

A2.

$$\begin{cases} m D^2 x_1 = f - k x_1 - b D x_1 + k(x_2 - x_1) \\ m D^2 x_2 = -k(x_2 - x_1) \end{cases}$$

$$\begin{cases} m D^2 x_1 = f - 2k x_1 - b D x_1 + k x_2 \\ m D^2 x_2 = -k x_2 + k x_1 \end{cases}$$

$$\begin{cases} k x_2 = m D^2 x_1 + b D x_1 + 2k x_1 - f \\ m D^2 x_2 + k x_2 = k x_1 \end{cases}$$

$$\begin{matrix} (mD^2+k) \\ k \end{matrix} \begin{cases} k x_2 = m D^2 x_1 + b D x_1 + 2k x_1 - f \\ (m D^2 + k) x_2 = k x_1 \end{cases}$$

$$(m D^2 + k)(m D^2 + b D + 2k) x_1 - (m D^2 + k) f = k^2 x_1$$

$$(m^2 D^4 + m b D^3 + 2 k m D^2 + k m D^2 + k b D + \cancel{2} k^2) x_1 - \cancel{k^2 x_1} = (m D^2 + k) f$$

eq. diff. $m^2 D^4 x_1 + m b D^3 x_1 + 3 k m D^2 x_1 + k b D x_1 + k^2 x_1 = m D^2 f + k f$

f.d.t. $G(s) = \dfrac{m s^2 + k}{m^2 s^4 + m b s^3 + 3 k m s^2 + k b s + k^2}$

3. Il guadagno statico è G(0)=1/k.

Gli zeri sono $z_{1,2} = \pm j\sqrt{\frac{k}{m}}$

B1.

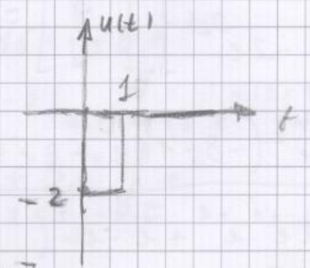
Vedi le dispense del corso.

B2.

Vedi le dispense del corso.

B3.

B3. $G(s) = \dfrac{8}{(s+1)(s+2)}$



Determinare $y(t)$ per $t > 0$

$u(t) = -2 \cdot 1(t)$ per $t \in [0,1]$

$U(s) = -2 \cdot \dfrac{1}{s}$ $\qquad Y(s) = -2 \cdot \dfrac{1}{s} \cdot \dfrac{8}{(s+1)(s+2)} =$

$$Y(s) = \frac{-16}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$K_1 = \left.\dfrac{-16}{(s+1)(s+2)}\right|_{s=0} = \dfrac{-16}{1 \cdot 2} = -8$ $\qquad K_2 = \left.\dfrac{-16}{s(s+2)}\right|_{s=-1} = \dfrac{-16}{(-1)\cdot 1} = +16$

$K_1 + K_2 + K_3 = 0 \qquad K_3 = 8 - 16 = -8$

$$y(t) = -8 + 16e^{-t} - 8e^{-2t} \qquad t \in [0,1]$$

$$Dy(t) = -16e^{-t} + 16e^{-2t}$$

$$Dy(0+) = -16 + 16 = 0 \qquad y(0+) = 0 \qquad \text{ok!}$$

$$y(t) = c_1 e^{-t} + c_2 e^{-2t} \quad \text{per } t \geq 1$$

$$Dy(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$$

$$y(t) \in \overline{C}^{g-1}, \text{ quindi essendo } g = 2$$

$$\Rightarrow y(t) \in \overline{C}^{1,\infty}$$

$$\begin{cases} y(1-) = y(1+) \\ Dy(1-) = Dy(1+) \end{cases}$$

$$\begin{cases} -8 + 16e^{-1} - 8e^{-2} = c_1 e^{-1} + c_2 e^{-2} \\ -16e^{-1} + 16e^{-2} = -c_1 e^{-1} - 2c_2 e^{-2} \end{cases}$$

$$\begin{bmatrix} e^{-1} & e^{-2} \\ -e^{-1} & -2e^{-2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -8 + 16e^{-1} - 8e^{-2} \\ -16e^{-1} + 16e^{-2} \end{bmatrix}$$

$$c_1 = \frac{-2e^{-2}\left(-8 + 16e^{-1} - 8e^{-2}\right) - \left(-16e^{-1} + 16e^{-2}\right)e^{-2}}{-2e^{-1} \cdot e^{-2} + e^{-1} \cdot e^{-2}} =$$

$$= \frac{-2\left(-8 + 16e^{-1} - 8e^{-2}\right) + 16e^{-1} - 16e^{-2}}{\underbrace{-2e^{-3} + e^{-3}}_{\left(\cancel{-e^{-3}}\right) \left(-e^{-1}\right)}} =$$

$$= \frac{16 - 32e^{-1} + \cancel{16e^{-2}} + 16e^{-1} - \cancel{16e^{-2}}}{-e^{-1}} = \frac{16 - 16e^{-1}}{-e^{-1}} =$$

$$= -16 \cdot e + 16 = 16 - 16 \cdot e$$

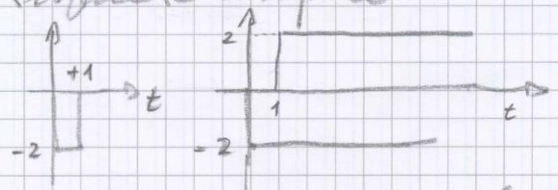
$$c_2 = \frac{\cancel{-16e^{-2}} + 16e^{-3} - 8e^{-1} \cancel{+16e^{-2}} - 8e^{-3}}{-e^{-3}} =$$

$$= \frac{-8e^{-1} + 8e^{-3}}{-e^{-3}} = 8e^{2} - 8$$

determinazione di $y(t)\big|_{t \geq 0}$ con le sole proprietà della trasformata di Laplace

$$\mathcal{L}[f(t-t_0)] = e^{-t_0 s}\mathcal{L}[f(t)]$$

$$f(t-t_0) = \mathcal{L}^{-1}\left[e^{-t_0 s}F(s)\right]$$



$$u(t) = -2 \cdot 1(t) + 2 \cdot 1(t-1)$$

$$U(s) = -2 \cdot \frac{1}{s} + 2 \cdot e^{-s} \cdot \frac{1}{s}$$

$$Y(s) = G(s)\,U(s) = \frac{-16}{s(s+1)(s+2)} + \frac{16}{s(s+1)(s+2)}e^{-s}$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{-16}{s(s+1)(s+2)}\right] + \mathcal{L}^{-1}\left[\frac{16}{s(s+1)(s+2)}e^{-s}\right] =$$

$$= \left(-8 + 16e^{-t} - 8e^{-2t}\right)\cdot 1(t) + \left(+8 - 16e^{-(t-1)} + 8e^{-2(t-1)}\right) 1(t-1)$$

Quindi per $t \in [0, 1)$: $y(t) = -8 + 16e^{-t} - 8e^{-2t}$

Per $t \geq 1$ $\quad y(t) = \cancel{-8} + 16e^{-t} - 8e^{-2t} \cancel{+8} - 16e^{-(t-1)} + 8e^{-2(t-1)} =$

$$= 16e^{-t} - 16e^{-t} \cdot e - 8e^{-2t} + 8e^{-2t} \cdot e^{2}$$

$$= (16 - 16e)e^{-t} + (-8 + 8e^{2})e^{-2t}$$

ok!

C1.

$$Y(s) = G(s)\frac{1}{s} = \frac{1}{s(s+2)(s+1-j)(s+1+j)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+1-j} + \frac{\overline{K}_3}{s+1+j}$$

$$K_1 = \left.\frac{1}{(s+2)[(s+1)^2+1]}\right|_{s=0} = \frac{1}{4}$$

$$K_2 = \left.\frac{1}{s[(s+1)^2+1]}\right|_{s=-2} = \frac{1}{(-2)[2]} = -\frac{1}{4}$$

$$K_3 = \left.\frac{1}{s(s+2)(s+1+j)}\right|_{s=-1+j} = \frac{1}{(-1+j)(-1+j+2)(-\cancel{1}+j+\cancel{1}+j)} =$$

$$= \frac{1}{(-1+j)(1+j)\,2j} = \frac{1}{[j^2-1]\,2j} = \frac{1}{(-2)\,2j} = \frac{1}{-4j}$$

$$K = \frac{j}{-4j^2} = \frac{1}{4}j \qquad |K_3| = \frac{1}{4} \quad \arg K_3 = +\frac{\pi}{2}$$

$$g_s(t) = \frac{1}{4} - \frac{1}{4}e^{-2t} + 2|K_3|\,e^{-t}\cdot\cos\left(t+\frac{\pi}{2}\right) =$$ ok!!

$$= \frac{1}{4} - \frac{1}{4}e^{-2t} + \frac{1}{2}e^{-t}\left[-\sin t\right] = \frac{1}{4} - \frac{1}{4}e^{-2t} - \frac{1}{2}e^{-t}\sin t$$

$$g_s(t) \in C^{\rho-1} \qquad \rho = 3 \qquad g_s(t) \in C^2$$

$$g(t) = D\,g_s(t) = \left(-\frac{1}{4}\right)(-2)e^{-2t} - \frac{1}{2}(-1)e^{-t}\sin t - \frac{1}{2}e^{-t}(\cos t) =$$

$$= \frac{1}{2}e^{-2t} + \frac{1}{2}e^{-t}\sin t - \frac{1}{2}e^{-t}\cos t$$ ok!

C2.

1. $G_{ry}(s)=\dfrac{L(s)}{1+L(s)}=\dfrac{\dfrac{16}{s(s+5)}}{1+\dfrac{16}{s(s+5)}}=\dfrac{16}{s(s+5)+16}=\dfrac{16}{s^2+5s+16}$

eq. diff.: $D^2y(t)+5Dy(t)+16y(t)=16r(t)$

2. Dal confronto $\dfrac{16}{s^2+5s+16}=\dfrac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$

$\omega_n=\sqrt{16}=4 \quad\Rightarrow\quad T_s\simeq\dfrac{1.8}{\omega_n}=0,45 \ \text{ sec.}$

$2\delta\omega_n=5 \ \Rightarrow\ \delta\omega_n=2.5 \ \Rightarrow\ T_a=\dfrac{3}{\delta\omega_n}=1,2 \ \text{ sec.}$

$\delta=\dfrac{2.5}{4}=0,625 \quad\Rightarrow\quad S=100\exp\left(-\dfrac{\delta\pi}{\sqrt{1-\delta^2}}\right)\cong 8{,}1\%$